

SusHi: A program for the calculation of Higgs production in gluon fusion and bottom-quark annihilation in the Standard Model and the MSSM

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Abstract

This article describes the code **SusHi** (for “**S**upersymmetric **H**iggs”) [1] which calculates the cross sections $pp/p\bar{p} \rightarrow \phi + X$ in gluon fusion and bottom-quark annihilation in the SM and the MSSM, where ϕ is any of the neutral Higgs bosons within these models. Apart from inclusive cross sections up to NNLO QCD, differential cross sections with respect to the Higgs’ transverse momentum p_T and (pseudo-)rapidity $y(\eta)$ can be calculated through NLO QCD. In case of gluon fusion, **SusHi** contains NLO QCD contributions from the third family of quarks and squarks, NNLO corrections due to top-quarks, approximate NNLO corrections due to top-squarks, and electro-weak effects. It supports various renormalization schemes for the sbottom sector and the bottom Yukawa coupling, as well as resummation effects of higher order $\tan\beta$ -enhanced sbottom contributions. **SusHi** provides a link to **FeynHiggs** for the calculation of the Higgs masses.

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1. Introduction

The recent observation of a new boson at the Large Hadron Collider (LHC) [2, 3] has opened a new chapter for Higgs phenomenology [4]. For the clear identification of this particle, precise predictions for Higgs production and decay will be absolutely essential. The current status of these efforts is collected in the reports of the LHC Higgs cross section working group [5, 6].

The main production mechanism for a Standard Model (SM) Higgs boson at a hadron collider is gluon fusion, where the gluon-Higgs coupling is mediated mostly by virtual top- and bottom-quarks ($\sim 7\%$). The total inclusive cross section is known through next-to-next-to-leading order (NNLO) in quantum chromodynamics (QCD) [7–18]. Even higher order QCD effects have been calculated [19–24] through resummation; electro-weak contributions reach up to 8% with respect to the leading order (LO) cross section [25–27].

The gluon-fusion mechanism for the neutral Higgs bosons in the Minimal Supersymmetric Standard Model (MSSM) is mediated by quarks and their superpartners, the squarks. For the CP-even MSSM Higgs bosons h, H , the QCD effects due to quarks can be simply taken over from the SM by a rescaling of the cross section with the corresponding modified Yukawa couplings. For the CP-odd Higgs boson A , the NNLO QCD corrections to the quark-induced total inclusive cross section have been calculated in Refs. [18, 28, 29].

Squark contributions to the gluon-Higgs coupling are typically suppressed by powers of $m_q/m_{\tilde{q}}$, and thus of importance mostly for small to moderate squark masses $m_{\tilde{q}}$. For the CP-odd Higgs boson A , they are absent at LO.¹

¹Comprehensive reviews of the Higgs theory within the SM and the MSSM can be found in Refs. [30, 31].

The individual components for a calculation of NLO QCD corrections in the MSSM are known: The NLO cross section due to (s)top (s)quarks, gluons, and gluinos was calculated for a CP-even Higgs boson with mass m_ϕ in Refs. [32–34] by applying an effective-theory approach in the limit $m_\phi \ll m_t, m_{\tilde{t}}, m_{\tilde{g}}$, similar to what is used in the SM at higher orders. In this approach, even NNLO effects have been first approximated [35], and recently fully calculated [36, 37]. The analogous NLO result for a CP-odd Higgs was first obtained in Ref. [38]. Due to the smallness of the bottom-quark mass, these results cannot be transferred to the bottom/sbottom sector. However, the limit $m_\phi, m_b \ll m_{\tilde{b}}, m_{\tilde{g}}$ is applicable in a large region of the parameter space, and the corresponding results were presented in Refs. [39, 40]. Recently, more general results for the cross sections, allowing for larger Higgs masses, were obtained for the CP-even and -odd Higgs bosons and both for the top/stop and the bottom/sbottom sector, in Refs. [41, 42]. A fully numerical calculation of the gluon-Higgs form factor for general quark/squark/gluino/Higgs-masses has been reported in Ref. [43]. For the pure squark contributions, the full Higgs-mass dependence of the NLO contribution to the cross section was presented (numerically and/or analytically) in Refs. [44–46].

For large values of the MSSM parameter $\tan\beta$, the coupling of the light CP-even Higgs boson to bottom-quarks is significantly enhanced relative to the SM Yukawa coupling, so the bottom sector may be much more important for gluon fusion in the MSSM. In addition, an enhanced bottom-Higgs coupling increases the cross section of another Higgs production mechanism in the MSSM, namely associated production with bottom-quarks, $pp/p\bar{p} \rightarrow b\bar{b}\phi$. If the final state quarks are not tagged, a suitable theoretical approach to the cross section is the process $b\bar{b} \rightarrow \phi$, called bottom-quark annihilation in what follows.² It resums terms of the form $\ln m_b/m_\phi$ by means of b -parton distribution functions (PDFs) and was calculated up to NNLO QCD in the SM [48, 49]. The result can be directly translated into the MSSM by rescaling it with the proper bottom-Yukawa coupling; even the dominant sbottom effects can be taken into account by an effective coupling [50, 51]. **SusHi** evaluates both the cross section for gluon fusion and bottom-quark annihilation.

For gluon fusion, **SusHi** includes results for all NLO QCD contributions due to the third generation of quarks and squarks. The real corrections at NLO are well-known; **SusHi** implements them using the routines of Ref. [40]. For the virtual corrections to the pure quark diagrams, it uses the analytic expression of Ref. [52] which was obtained from the integral representation in Ref. [10]. Concerning the genuine virtual supersymmetric (SUSY) corrections, it employs the results of Refs. [32, 33, 41, 42] and Refs. [39, 41] for the (s)top- and the (s)bottom-mediated gluon-Higgs coupling, respectively. NNLO QCD effects are taken into account for the top-quark induced gluon-Higgs coupling [11–13, 28, 29], and approximately for the top/stop/gluino-induced one [35] by using **ggh@nnlo** [53]. Electroweak corrections [25–27] are included as tabulated correction factors. The cross section is provided in various renormalization schemes (in particular in the sbottom sector), allowing for an on-shell, $\overline{\text{DR}}$, or a dependent renormalization of the soft-breaking parameter A_b , for example. In addition, the bottom-Yukawa coupling can be chosen on-shell or in the $\overline{\text{MS}}$ -scheme. Higher-order sbottom effects can be included through the parameter Δ_b [54–59].

²See Ref. [47] for a more detailed discussion.

For the calculation of the bottom-quark annihilation cross section, **SusHi** makes use of **bbh@nnlo** [60] and re-weights its results by the MSSM couplings. It uses the **LHAPDF** library [61] which allows to conveniently switch between different PDF sets, and it can be linked to **FeynHiggs** [62–65] for the two-loop calculation of the Higgs boson masses in the MSSM.

Apart from inclusive cross sections for gluon fusion and bottom-quark annihilation, **SusHi** allows for (upper and lower) cuts on the transverse momentum and/or the (pseudo-)rapidity of the outgoing scalar ϕ . In case of gluon fusion, differential distributions with respect to these kinematic variables can be obtained (for p_T not too small).

Note that a number of codes for the calculation of Higgs cross sections in the SM and the MSSM exist, see Refs. [66–72], for example. They overlap with **SusHi** to a greater or lesser extent; the distinctive feature of **SusHi** is to provide full NLO QCD (and partial NNLO and electro-weak) corrections for the dominant production mechanisms of the three neutral Higgs bosons of the MSSM, both inclusive and differential, in various renormalization schemes. Further details will be given below.

The remainder of this paper is organized as follows: In Section 2, we present the physical background of **SusHi**, recalling the framework of the Higgs and quark/squark sectors in the MSSM with special emphasis on the renormalization of the squark sectors and the resummation of $\tan\beta$ -enhanced sbottom corrections in the bottom Yukawa coupling. Subsequently, we discuss the various contributions for the calculation of the gluon-fusion and the bottom-quark annihilation cross section as they enter in **SusHi**. We briefly describe the kinematic variables for which cuts can be applied and distributions be obtained. In Section 6, we describe the program **SusHi**, in particular its workflow, installation, and usage, as well as the input and output files. Our conclusions are given in Section 7. Appendix A contains the couplings of the squarks to the Higgs bosons ϕ .

2. Physics background

This section first introduces our notation for the relevant parts of the SM and the MSSM. It describes the renormalization of the squark sector and the possible choices for the bottom Yukawa coupling provided in **SusHi**.

2.1. Standard Model

The SM contains one scalar weak isospin doublet which corresponds to a single physical particle H (electric charge and spin zero). The pure Higgs sector of the Lagrangian is determined by two parameters: the vacuum expectation value $v \approx 246 \text{ GeV}$, and the mass of the Higgs boson m_H . The Yukawa couplings of the fermions to the Higgs boson are given by $Y_f = \sqrt{2}m_f/v$, where m_f is the fermion mass. **SusHi** requires all input masses in the on-shell scheme, except for the charm- and bottom-quark mass which has to be given as the $\overline{\text{MS}}$ mass $m_q(m_q) \equiv m_q^{\overline{\text{MS}}}(m_q^{\overline{\text{MS}}})$, $q \in \{c, b\}$. The calculation of various internal bottom masses is addressed in Section 2.3.

2.2. Supersymmetry

The MSSM contains two Higgs doublets, named H_d and H_u , which develop the vacuum expectation values $v_d = v \cos \beta$ and $v_u = v \sin \beta$, where the parameter β is undetermined. They form two CP-even Higgs fields h, H , one CP-odd (or “pseudo-scalar”) Higgs field A , and two charged Higgs fields H^\pm . At lowest order, the mass spectrum of the Higgs sector is determined by SM parameters, $\tan \beta = v_u/v_d$, and the CP-odd Higgs mass m_A . Radiative corrections to the Higgs mass spectrum are generally quite large [73–75]; currently, they are known through three-loop order [76–78]. **SusHi** provides a way to conveniently take into account two-loop corrections by linking to the program **FeynHiggs** [62–65]. This is an optional feature, however; any Higgs mass can be given as an input to **SusHi**.

The mixing of the isospin to the Higgs mass eigenstates in the CP-even sector is governed by the angle α . Together with β , it determines the relative strengths of the Higgs boson couplings g_f^ϕ ($\phi \in \{h, H, A\}$) to the SM fermions with respect to the SM Higgs boson couplings (see Appendix A), which thus enter the fermion Yukawa couplings $Y_f^\phi = \sqrt{2}m_f g_f^\phi/v$:

$$\begin{aligned} g_u^h &= \frac{\cos \alpha}{\sin \beta}, & g_u^H &= \frac{\sin \alpha}{\sin \beta}, & g_u^A &= \frac{1}{\tan \beta}, \\ g_d^h &= -\frac{\sin \alpha}{\cos \beta}, & g_d^H &= \frac{\cos \alpha}{\cos \beta}, & g_d^A &= \tan \beta. \end{aligned} \quad (1)$$

These normalized couplings are independent of the fermion generation. Our calculation includes the third generation of squarks which enters the Lagrangian in the form

$$\mathcal{L} \supset -(\tilde{q}_L^\dagger, \tilde{q}_R^\dagger) \mathcal{M}_{\tilde{q}}^2 \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \quad (2)$$

with the mass matrix

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} M_L^2 + m_q^2 + m_Z^2 \cos(2\beta)(T_q^3 - Q_q s_W^2) & m_q(A_q - \mu \kappa_q) \\ m_q(A_q - \mu \kappa_q) & M_{\tilde{q}R}^2 + m_q^2 + m_Z^2 \cos(2\beta)Q_q s_W^2 \end{pmatrix} \quad (3)$$

for an arbitrary species of squarks \tilde{q} . This formula contains the SUSY soft-breaking parameters M_L^2 , $M_{\tilde{q}R}^2$, and A_q . The parameter μ determines the mass of the fermionic Higgs partners, the Higgsinos, whereas the Z -boson mass m_Z and the weak mixing angle θ_W ($s_W = \sin \theta_W$) are the usual SM parameters; m_q , Q_q , and T_q^3 are the mass, the electric, and the weak charge of the corresponding quark q , respectively. It is $\kappa_b = \tan \beta =: t_\beta$ and $\kappa_t = 1/t_\beta$.

The physical particle states are obtained by the diagonalization of the mass matrix in Eq. (3) which we do in accordance with Ref. [79] using

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = U_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} \quad \text{with} \quad U_{\tilde{q}} = \begin{pmatrix} \cos \theta_{\tilde{q}} & \sin \theta_{\tilde{q}} \\ -\sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}} \end{pmatrix}. \quad (4)$$

By choosing $0 \leq \theta_q < \pi$, the masses of the squarks \tilde{q}_1 and \tilde{q}_2 are ordered $m_{\tilde{q}_1} < m_{\tilde{q}_2}$ and given by the square roots of the eigenvalues of $\mathcal{M}_{\tilde{q}}^2$ in Eq. (3):

$$\begin{aligned} m_{\tilde{q}_{12}}^2 &= \frac{1}{2}(M_L^2 + M_{\tilde{q}R}^2) + m_q^2 + \frac{1}{2}T_q^3 m_Z^2 \cos(2\beta) \\ &\mp \frac{1}{2}\sqrt{(M_L^2 + M_{\tilde{q}R}^2 + m_Z^2 \cos(2\beta)(T_q^3 - 2Q_q \sin^2 \theta_W))^2 + 4m_q^2(A_q - \mu \kappa_q)^2}. \end{aligned} \quad (5)$$

The entries of $\mathcal{M}_{\tilde{q}}^2$ can also be expressed in terms of the mass eigenvalues and the mixing angle

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} m_{\tilde{q}1}^2 \cos^2 \theta_{\tilde{q}} + m_{\tilde{q}2}^2 \sin^2 \theta_{\tilde{q}} & (m_{\tilde{q}1}^2 - m_{\tilde{q}2}^2) \sin \theta_{\tilde{q}} \cos \theta_{\tilde{q}} \\ (m_{\tilde{q}1}^2 - m_{\tilde{q}2}^2) \sin \theta_{\tilde{q}} \cos \theta_{\tilde{q}} & m_{\tilde{q}1}^2 \sin^2 \theta_{\tilde{q}} + m_{\tilde{q}2}^2 \cos^2 \theta_{\tilde{q}} \end{pmatrix}, \quad (6)$$

which implies that the mixing angle can be obtained from

$$\sin(2\theta_{\tilde{q}}) = \frac{2m_q(A_q - \mu\kappa_q)}{m_{\tilde{q}1}^2 - m_{\tilde{q}2}^2}. \quad (7)$$

Note that Eq. (7) does not uniquely define $\theta_{\tilde{q}}$; a shift $\theta_{\tilde{q}} \rightarrow \frac{\pi}{2} - \theta_{\tilde{q}}$, which corresponds to $\sin \theta_{\tilde{q}} \leftrightarrow \cos \theta_{\tilde{q}}$, might be in order to allow for $m_{\tilde{q}1} < m_{\tilde{q}2}$. For completeness, the couplings of the squarks to the MSSM Higgs bosons can be found in Appendix A.

2.2.1. Renormalization of the (s)top sector

Using Eq. (7), we eliminate A_t from the (s)top sector contribution of the amplitude before renormalization and express it in terms of the on-shell parameters for the top mass m_t , the stop masses $m_{\tilde{t}1}$ and $m_{\tilde{t}2}$ and the mixing angle $\theta_{\tilde{t}}$, defined according to Section 3.1 in Ref. [79]. In practice, the user specifies the soft-breaking parameters $M_L \equiv M_L(\tilde{t})$, $M_{\tilde{t}R}$, and A_t , as well as the on-shell top-quark mass m_t^{OS} . Setting $m_t = m_t^{\text{OS}}$, **SusHi** inserts them into the mass matrix of Eq. (3) whose eigenvalues $m_{\tilde{t}1}^2, m_{\tilde{t}2}^2$, as well as the corresponding stop mixing angle θ_t are interpreted as on-shell parameters.³ Note that in case **FeynHiggs** is used, the on-shell stop masses and the stop mixing angle are simply taken over from its output.

2.2.2. Renormalization of the (s)bottom sector

The renormalization of the (s)bottom sector is more subtle. At tree-level, the soft-breaking parameter M_L^2 is identical for the sbottom and stop sector due to $SU(2)_L$ symmetry. At higher orders, however, in the on-shell scheme we distinguish $M_L^2(\tilde{t})$ and $M_L^2(\tilde{b})$ similar to Ref. [80–83] by

$$M_L^2(\tilde{b}) = M_L^2(\tilde{t}) + \delta M_L^2(\tilde{t}) - \delta M_L^2(\tilde{b}) \equiv M_L^2(\tilde{t}) + \Delta M_L^2, \quad (8)$$

with the individual counterterms given by

$$\delta M_L^2(\tilde{q}) = \cos^2 \theta_{\tilde{q}} \delta m_{\tilde{q}1}^{2,\text{OS}} + \sin^2 \theta_{\tilde{q}} \delta m_{\tilde{q}2}^{2,\text{OS}} - (m_{\tilde{q}1}^2 - m_{\tilde{q}2}^2) \sin(2\theta_{\tilde{q}}) \delta \theta_{\tilde{q}}^{\text{OS}} - 2m_q \delta m_q^{\text{OS}}, \quad (9)$$

where the on-shell counterterms $\delta m_{\tilde{q}1}^{2,\text{OS}}$, $\delta m_{\tilde{q}2}^{2,\text{OS}}$, δm_q^{OS} , and $\delta \theta_{\tilde{q}}^{\text{OS}}$ are analogously defined as in the top sector, see Ref. [79].

Note that the finite shift ΔM_L^2 depends on $m_{\tilde{b}1}, m_{\tilde{b}2}$, and θ_b . In order to determine its numerical value, we first calculate “tree-level” values for the sbottom masses and mixing angle by inserting the parameters

$$M_L^2 \equiv (M_L^2(\tilde{b}))^{\text{tree}} = M_L^2(\tilde{t}), \quad M_{\tilde{b}R}^2, \quad A_b, \quad m_b^{\text{OS}} \quad (10)$$

³This can be seen as an indirect definition of the renormalization scheme for M_L , $M_{\tilde{t}R}$, and A_t .

into the mass matrix (3) (setting $m_b = m_b^{\text{OS}}$). All parameters of Eq. (10) are input to **SusHi**, except for the on-shell bottom-quark mass m_b^{OS} which is determined from the input parameter $m_b(m_b)$ as described in Section 2.3. With these tree-level sbottom masses we define the scale

$$\mu_d = \frac{1}{3}(m_{\tilde{g}} + m_{\tilde{b}_1} + m_{\tilde{b}_2}). \quad (11)$$

As was pointed out in Refs. [39, 79, 84, 85], replacing A_b in the amplitude through Eq. (7) before renormalization – analogous to the stop sector – leads to potentially large corrections $\delta A_b \propto (\alpha_s \mu^2 \tan^2 \beta)/m_{\tilde{g}}$. It was therefore suggested to use Eq. (7) in order to eliminate m_b . The counterterm for m_b in this scheme, denoted δm_b^{dep} , is obtained from Eq. (7):

$$\delta m_b^{\text{dep}} = 2m_b \cot(2\theta_b) \delta\theta_b - \frac{2m_b^2 \cdot \delta A_b}{\sin(2\theta_b)(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)} + m_b \frac{\delta m_{\tilde{b}_1}^{2,\text{OS}} - \delta m_{\tilde{b}_2}^{2,\text{OS}}}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}. \quad (12)$$

Here it is already implied that we always renormalize the sbottom masses on-shell, while the renormalization of A_b and θ_b is still unspecified.

In order to calculate the on-shell sbottom masses, we choose the “on-shell” renormalization of A_b , defined through a kinematical condition on the $A\tilde{b}_1\tilde{b}_2$ -vertex [39, 79, 84, 85]. The corresponding counterterm is

$$\delta A_b^{\text{OS}} = (A_b + \mu \cot \beta) \left[f(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) + f(m_{\tilde{b}_2}^2, m_{\tilde{b}_1}^2) - \frac{\delta m_b^{\text{dep}}}{m_b} \right], \quad (13)$$

with

$$f(m_1^2, m_2^2) = -\frac{\alpha_s(\mu_d)}{\pi} \frac{2}{3} \left\{ \left[-\frac{m_{\tilde{g}}}{A_b + \mu \cot \beta} B_0^{\text{fin}}(m_1^2, m_b, m_{\tilde{g}}, \mu_d) \right] + \frac{m_1^2}{m_1^2 - m_2^2} \left[4 + 2 \log \frac{\mu_R^2}{m_1^2} - \left(1 - \frac{m_{\tilde{g}}^2}{m_1^2} - \frac{m_b^2}{m_1^2} \right) \cdot B_0^{\text{fin}}(m_1^2, m_b, m_{\tilde{g}}, \mu_d) \right] \right\}, \quad (14)$$

where the function B_0^{fin} can be taken from Eq. (B.8) in Ref. [33] replacing $\mu_R \rightarrow \mu_d$. Solving Eq. (12) (with $\delta A_b = \delta A_b^{\text{OS}}$) and Eq. (13) for δA_b^{OS} , we obtain

$$\begin{aligned} \delta A_b^{\text{OS}} = & (A_b \sin \beta + \mu \cos \beta) \left\{ -(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2) \sin(2\theta_b) [f(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) + f(m_{\tilde{b}_2}^2, m_{\tilde{b}_1}^2)] \right. \\ & \left. + (\delta m_{\tilde{b}_1}^{2,\text{OS}} - \delta m_{\tilde{b}_2}^{2,\text{OS}}) \sin(2\theta_b) + 2\delta\theta_b(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2) \cos(2\theta_b) \right\} \\ & \cdot [2(\sin \beta (A_b m_b - (m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2) \sin \theta_b \cos \theta_b) + m_b \mu \cos \beta)]^{-1}, \end{aligned} \quad (15)$$

which in turn allows to calculate δm_b^{dep} . Note that one is still free to choose the renormalization condition for θ_b .

The on-shell sbottom masses are finally obtained as follows: We calculate the counterterm δm_b^{dep} from Eq. (12) using $\delta A_b = \delta A_b^{\text{OS}}$ and $\delta\theta_b = \delta\theta_b^{\text{OS}}$, the “tree-level” sbottom masses and mixing angle and the on-shell bottom mass. This yields a numerical value for ΔM_L^2 and similarly for $m_b^{\text{dep}} = m_b^{\text{DR}}(\mu_d) - \delta m_b^{\text{dep}} + \delta m_b^{\text{DR}}$, where the $\overline{\text{DR}}$ mass is calculated in the SM.

Diagonalizing the mass matrix \mathcal{M}_b^2 in Eq. (3) with $m_b = m_b^{\text{dep}}$, we thus obtain on-shell sbottom masses $m_{\tilde{b}1}$ and $m_{\tilde{b}2}$ and the on-shell sbottom mixing angle $\theta_{\tilde{b}}$. In case **FeynHiggs** is called for the calculation of the MSSM Higgs masses, we take m_b^{dep} as well as ΔM_L^2 from this program which, when inserted into the sbottom mass matrix Eq. (3), results in on-shell sbottom masses and mixing angle consistent with the values given by **FeynHiggs** itself.

After the on-shell sbottom masses have been determined, **SusHi** allows for a change between various schemes regarding the renormalization of m_b , A_b , and $\theta_{\tilde{b}}$ in the sbottom contribution to the gluon-fusion amplitude. Note, however, that either m_b or A_b is required to be a dependent quantity; a dependent renormalization of the sbottom mixing angle is not offered as option. The possible choices are summarized in Tab. 1. Numerical differences in the cross sections between the various renormalization schemes and their implications will be investigated in a separate publication.

Scheme choices			
m_b	dep.	OS	$\overline{\text{DR}}$
A_b	dep.	OS	$\overline{\text{DR}}$
$\theta_{\tilde{b}}$		OS	$\overline{\text{DR}}$

Table 1: Available renormalization schemes for the sbottom sector. The default option is marked in red.

To summarize, apart from μ , $\tan\beta$ and the SM parameters, the input parameters of **SusHi** that determine the squark sectors are:

- M_L^2 , M_{iR}^2 , A_t , m_t^{OS} which directly determine the on-shell stop masses and mixing angle through diagonalization of Eq. (3)
- M_{bR}^2 , A_b , $m_b(m_b)$, where A_b is understood as renormalized according to Eq. (13).

Switching between renormalization schemes changes both the counterterms to the amplitude as well as the numerical values of the parameters m_b , $m_{\tilde{b}1}$, $m_{\tilde{b}2}$, A_b , and $\theta_{\tilde{b}}$, of course. We remark that changing the renormalization scheme in the sbottom sector affects m_b only in the Higgs-sbottom couplings; the renormalization of the bottom mass occurring in the Higgs-bottom Yukawa coupling is independent of that (see Section 2.3). For the bottom mass occurring in internal propagators (rather than in couplings), **SusHi** always uses the on-shell value.

The switching between the different renormalization schemes is done at the renormalization scale μ_R to guarantee the same coupling strength $\alpha_s(\mu_R)$ for the NLO counterterms and the cross section itself. In case **SusHi** makes use of the formulas in Refs. [39, 41], the counterterms at NLO are expanded to the correct order in the bottom mass to match the expanded NLO amplitudes.

2.3. Bottom mass calculation/Resummation of $\tan\beta$ -enhanced contributions

The bottom-quark mass in the input files of **SusHi** is inserted in the $\overline{\text{MS}}$ scheme $m_b(m_b)$. Together with the input value of $\alpha_s(m_Z)$, we calculate $\alpha_s(m_b(m_b))$ by 4-loop running with 5 active flavors. Using Eq. (13) of Ref. [86] at 3-loop level (see also Ref. [87]), $m_b(m_b)$ is transformed into its on-shell value m_b^{OS} .

In the SM, the bottom Yukawa coupling $Y_b = \sqrt{2}m_b^Y/v$ can be chosen $m_b^Y = m_b^{\text{OS}}$ or alternatively $m_b^Y = m_b^{\overline{\text{MS}}}(\mu_b)$, where $\mu_b \in \{m_b, \mu_R\}$; μ_R denotes the renormalization scale. As indicated above, the bottom mass entering the internal propagators is always set to the on-shell mass m_b^{OS} in **SusHi**. In the MSSM, **SusHi** offers various options regarding the choice of the Higgs-bottom Yukawa coupling $Y_b^\phi = \sqrt{2}m_b^{Y,\phi}g_f^\phi/v$:

$$\text{on-shell coupling: } m_b^{Y,\phi} = m_b^{\text{OS}} \quad (16a)$$

$$\text{basic resummation: } m_b^{Y,\phi} = \frac{m_b^{\text{OS}}}{1 + \Delta_b} \quad (16b)$$

$$\text{full resummation } h: m_b^{Y,h} = \frac{m_b^{\text{OS}}}{1 + \Delta_b} \left(1 - \Delta_b \frac{\cot\alpha}{\tan\beta} \right) \quad (16c)$$

$$H: m_b^{Y,H} = \frac{m_b^{\text{OS}}}{1 + \Delta_b} \left(1 + \Delta_b \frac{\tan\alpha}{\tan\beta} \right) \quad (16d)$$

$$A: m_b^{Y,A} = \frac{m_b^{\text{OS}}}{1 + \Delta_b} \left(1 - \Delta_b \frac{1}{\tan^2\beta} \right) \quad (16e)$$

$$\text{Running coupling: } m_b^{Y,\phi} = \frac{m_b^{\overline{\text{MS}}}(\mu_b)}{1 + \Delta_b} \quad (16f)$$

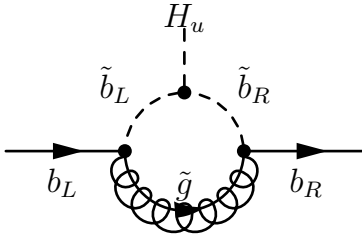


Figure 1: Feynman diagram inducing an effective coupling of the bottom-quarks to H_u . The coupling is proportional to μt_β and can be resummed in Δ_b .

Herein, Δ_b resums higher order sbottom contributions as shown in Fig. 1 [54–59], for example, calculated at the scale μ_d , defined in Eq. (11). The exact formula is given by:

$$\Delta_b = \frac{2}{3\pi} \alpha_s(\mu_d) m_{\tilde{g}} \mu t_\beta I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \quad (17)$$

$$I(a, b, c) = \frac{ab \ln(\frac{a}{b}) + bc \ln(\frac{b}{c}) + ca \ln(\frac{c}{a})}{(a-b)(b-c)(a-c)} \quad (18)$$

If the input values for **SusHi** are determined by **FeynHiggs**, **SusHi** allows to use the value of Δ_b as given in the output of this program which contains also the electro-weak contributions from neutralinos and charginos in accordance with Ref. [88]; two-loop corrections to Δ_b [89–91] are not yet included. For the running coupling in Eq. (16f), the scale μ_b can be set to m_b or μ_R . The numerical differences between the various schemes will be discussed in a forthcoming publication.

3. Cross section for gluon fusion

As indicated in the Introduction, the most important production channel in the SM and for moderate values of t_β in the MSSM is gluon fusion. After quoting the well-known results for the LO cross section, our implementation of the NLO contributions is explained. A discussion of NNLO and electro-weak contributions follows.

Using the notation of Ref. [10], the hadronic cross section for $\phi \in \{h, H, A\}$ at NLO QCD can be written as follows:

$$\sigma(pp \rightarrow \phi + X) = \sigma_0^\phi \left[1 + C^\phi \frac{\alpha_s}{\pi} \right] \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi} + \Delta\sigma_{gg}^\phi + \Delta\sigma_{gq}^\phi + \Delta\sigma_{q\bar{q}}^\phi, \quad (19)$$

where $\tau_\phi = m_\phi^2/s$, with the hadronic center-of-mass energy s . The factor σ_0^ϕ is determined by the LO cross section, C^ϕ arises from NLO terms in the partonic cross section that are singular as $\hat{s} \rightarrow m_\phi^2$ (\hat{s} is the partonic center-of-mass energy), and

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x) g(\tau/x) \quad (20)$$

is the gluon-gluon luminosity. The quantities $\Delta\sigma_{gg}^\phi$, $\Delta\sigma_{gq}^\phi$, and $\Delta\sigma_{q\bar{q}}^\phi$ comprise the terms that are regular as $\hat{s} \rightarrow m_\phi^2$ in the partonic cross section, arising from gg , gq and $q\bar{q}$ scattering, respectively. Loosely speaking, C^ϕ is due to the virtual, while the $\Delta\sigma_{ij}^\phi$ are due to the real radiation contributions. The latter are implemented in **SusHi** by expressing them in terms of Passarino-Veltman functions [92], see Ref. [40]. The strong coupling α_s for the cross section calculations is renormalized in standard QCD with five active quark flavors.

3.1. LO cross section

Taking into account the third generation of quarks (and squarks in the MSSM), the normalization factor in Eq. (19) can be written in the form

$$\sigma_0^\phi = \frac{G_F \alpha_s^2(\mu_R)}{288\sqrt{2}\pi} |\mathcal{A}^{\phi,(0)}|^2, \quad (21)$$

with Fermi's constant G_F . The amplitude \mathcal{A} for is given by

$$\mathcal{A}^{\phi,(0)} = \sum_{q \in \{t, b\}} (a_q^{\phi,(0)} + \tilde{a}_q^{\phi,(0)}) , \quad (22)$$

with the individual contributions for $\phi \in \{h, H\}$

$$a_q^{\phi,(0)} = g_q^\phi \frac{3\tau_q^\phi}{2} (1 + (1 - \tau_q^\phi)f(\tau_q^\phi)), \quad \tilde{a}_q^{\phi,(0)} = -\frac{3\tau_q^\phi}{8} \sum_{i=1}^2 g_{\tilde{q},ii}^\phi (1 - \tau_{\tilde{q}i}^\phi f(\tau_{\tilde{q}i}^\phi)), \quad (23)$$

using the notation

$$\tau_q^\phi = \frac{4m_q^2}{m_\phi^2}, \quad \tau_{\tilde{q}i}^\phi = \frac{4m_{\tilde{q}i}^2}{m_\phi^2} \quad (24)$$

and the function

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left(\log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right)^2 & \tau < 1 \end{cases}. \quad (25)$$

For the CP-odd Higgs $\phi = A$, the squarks do not contribute at LO, i.e. $\tilde{a}_q^{A,(0)} = 0$, and the quark contribution can be written in the form

$$a_q^{A,(0)} = g_q^A \frac{3\tau_q^A}{2} \tau_q^A f(\tau_q^A). \quad (26)$$

The couplings g_f^ϕ of the Higgs ϕ to the quarks can be taken from Eq. (1), the couplings $g_{\tilde{q},ij}^\phi$ to the squarks from Appendix A. Needless to say, in the SM, the squark couplings have to be set to zero and the quark couplings to $g_q^\phi = 1$.

3.2. NLO virtual contributions

As indicated above, the coefficient C^ϕ contains the virtual corrections to the gg process and is regularized by the infrared singular part; moreover, it includes the counterterms to LO quantities. We write it as

$$C^\phi = 2\text{Re} \left[\frac{\mathcal{A}^{\phi,(1)}}{\mathcal{A}_\infty^{\phi,(0)}} \right] + \pi^2 + \beta_0 \log \left(\frac{\mu_R^2}{\mu_F^2} \right), \quad (27)$$

where $\beta_0 = 11/2 - n_f/3$ with $n_f = 5$; μ_F and μ_R denote the factorization and the renormalization scale, respectively. The NLO amplitude $\mathcal{A}^{\phi,(1)}$ and the LO amplitude $\mathcal{A}_\infty^{\phi,(0)}$ in the limit of large stop and sbottom masses are given by

$$\mathcal{A}^{\phi,(1)} = \sum_{q \in \{t,b\}} (a_q^{\phi,(1)} + \tilde{a}_q^{\phi,(1)}), \quad \mathcal{A}_\infty^{\phi,(0)} = \sum_{q \in \{t,b\}} \left(a_q^{\phi,(0)} + \frac{\tau_q^\phi}{8} \sum_{i=1}^2 \frac{g_{\tilde{q},ii}^\phi}{\tau_{\tilde{q}i}^\phi} \right). \quad (28)$$

Available results for the NLO contributions have been discussed in the Introduction. In **SusHi**, we use the analytic formulas of Ref. [52] for the quark-induced terms $a_q^{\phi,(1)}$. The purely squark-induced terms (see Fig. 2 (a), for example) need to be considered in combination with the mixed quark/squark/gluino diagrams (two examples are shown in Fig. 2) in order to preserve supersymmetry, resulting in the coefficient $\tilde{a}_q^{\phi,(1)}$. **SusHi** implements expansions for these amplitudes in two limits:

- $m_\phi \ll m_q, m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$ [32–34] which is valid through $m_\phi < \min(2m_q, 2m_{\tilde{q}}, m_{\tilde{q}} + m_q + m_{\tilde{g}})$ and thus applies to the top-stop sector as long as ϕ is not too heavy. **SusHi** incorporates the publicly available program `evalcsusy.f` [93] in order to use this result for the light Higgs h .
- $m_\phi, m_q \ll m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$ [39–42] which holds through $m_\phi < \min(2m_{\tilde{q}}, m_{\tilde{q}} + m_{\tilde{g}})$, and thus applies to the bottom-sbottom sector, as well as to the top-stop sector when ϕ is heavy; **SusHi** uses the formulas of Refs. [41, 42] in these cases.

Note that both expansions hold only as long as the Higgs mass is not much heavier than the typical SUSY mass. For larger Higgs masses, only the fully numerical result of Ref. [43] is known so far.

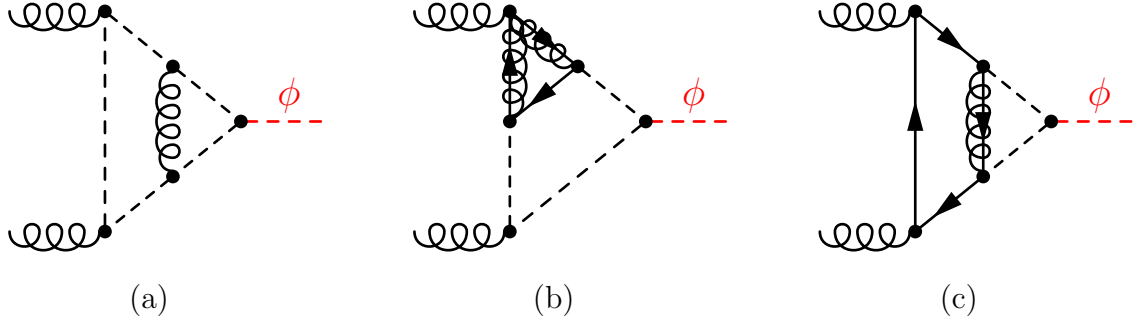


Figure 2: Example Feynman diagrams contributing to $\tilde{a}_q^{\phi,(1)}$: gluon-squark contribution (left), gluino-quark-squark contribution (middle) and gluino-quark-squark contribution (right). The latter is partially resummed by the usage of Δ_b from Eq. (17).

3.3. NNLO corrections

While the implementation of NLO corrections allows for the evaluation of inclusive and exclusive quantities, NNLO corrections are available in **SusHi** only for the total inclusive cross section. In addition, only the top-(s)quark induced gluon-Higgs coupling is taken into account at NNLO; the top-squark induced one only approximately:

$$\sigma_{gg\phi,\text{NNLO}}^{\text{MSSM}} = \sigma_{gg\phi,\text{NLO}}^{\text{MSSM}} + (\sigma_{gg\phi,\text{NNLO}}^t - \sigma_{gg\phi,\text{NLO}}^t), \quad (29)$$

where, on the right-hand side, the NNLO term is evaluated with the NNLO PDFs, while the NLO terms are evaluated as usual with NLO PDFs.

The (N)NLO top-(s)quark contributions to the cross sections $\sigma_{gg\phi,(N)\text{NLO}}^t$ are calculated with the help of the programs **ggh@nnlo** [53] and **evalcsusy** [93] which work in the effective theory approach of heavy top (s)quarks. The NNLO top-squark (and mixed top/stop/gluino) effects, available for $\phi = h$ only, have been evaluated [36, 37] by applying the limit $m_\phi \ll m_q, m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$. The result has been implemented in a computer code that involves **Mathematica** and a number of other programs. **SusHi** includes an approximation of these NNLO effects according to Ref. [35]⁴.

3.4. Electro-weak corrections

The full NLO electro-weak (EW) corrections are known only in the SM [25]. It has been suggested to assume complete factorization of QCD and EW effects [94], thus writing

$$\sigma_{ggH,\text{NNLO,EW}}^{\text{SM},t} = (1 + \delta_{\text{EW}}) \sigma_{ggH,\text{NNLO}}^{\text{SM},t}. \quad (30)$$

⁴The NNLO term of the normalized Wilson coefficient (κ_2 in the notation of Ref. [35]) is replaced by its SM value in **SusHi**.

For the CP-even Higgs bosons in the MSSM, a formula based on Eq. (29) and Eq. (30) has been used for the combination of QCD and electro-weak corrections in Ref. [40]:

$$\sigma_{gg\phi, \text{NNLO}, \text{EW}}^{\text{MSSM}} = \sigma_{gg\phi, \text{NLO}}^{\text{MSSM}} + (1 + \delta_{\text{EW}}) \sigma_{gg\phi, \text{NNLO}}^t - \sigma_{gg\phi, \text{NLO}}^t, \quad (31)$$

or, at NLO precision,

$$\sigma_{gg\phi, \text{NLO}, \text{EW}}^{\text{MSSM}} = \sigma_{gg\phi, \text{NLO}}^{\text{MSSM}} + \delta_{\text{EW}} \sigma_{gg\phi, \text{NLO}}^t. \quad (32)$$

Alternatively, it has been suggested in Ref. [67] to use the SM electro-weak corrections due to light quarks only [26, 27]. Following Ref. [67], we define the correction factor

$$\delta_{\text{EW}}^{\text{lf}} = \frac{\alpha_{\text{EM}}}{\pi} \frac{2\text{Re}(\mathcal{A}^{\phi, (0)} \mathcal{A}^{\phi, \text{EW}})}{|\mathcal{A}^{\phi, (0)}|^2}, \quad (33)$$

where $\mathcal{A}^{\phi, (0)}$ denotes the complete LO amplitude including quark and squark diagrams, see Eq. (22), and the electro-weak amplitude is given by [27]

$$\mathcal{A}^{\phi, \text{EW}} = -\frac{3}{8} \frac{x_W}{s_W^2} \left[\frac{2}{c_W^2} \left(\frac{5}{4} - \frac{7}{3} s_W^2 + \frac{22}{9} s_W^4 \right) A_1[x_Z] + 4A_1[x_W] \right] g_V^\phi, \quad (34)$$

with

$$x_V = \frac{1}{m_\phi^2} \left(m_V - i \frac{\Gamma_V}{2} \right)^2, \quad V \in \{W, Z\}, \quad (35)$$

the electro-magnetic coupling α_{EM} , and $s_W = \sin \theta_W = (1 - c_W^2)^{1/2}$ the sine of the weak mixing angle. Supersymmetry enters through the relative couplings g_V^ϕ given by

$$g_V^h = \sin(\beta - \alpha), \quad g_V^A = 0, \quad g_V^H = \cos(\beta - \alpha). \quad (36)$$

The function $A_1[x]$ can be found in Refs. [26, 27]. Since its numerical evaluation is rather involved, **SusHi** implements $\delta_{\text{EW}}^{\text{lf}}$ (and δ_{EW}) in terms of an interpolation grid in m_ϕ (and m_t), using fixed values for the gauge boson masses and widths, as well as for the weak mixing angle ($m_W = 80.385 \text{ GeV}$, $\Gamma_W = 2.085 \text{ GeV}$, $\sin^2 \theta_W = 0.22295$, $m_Z = 91.1876 \text{ GeV}$, $\Gamma_Z = 2.4952 \text{ GeV}$ [95]); the input values to **SusHi** for these parameters are ignored in the evaluation of the electro-weak corrections. The electro-weak correction factor due to light quarks $\delta_{\text{EW}}^{\text{lf}}$ multiplies the NLO MSSM cross section, while the NNLO QCD effects are simply added as in Eq. (29):

$$\sigma_{gg\phi}^{\text{MSSM}} = \sigma_{gg\phi, \text{NLO}}^{\text{MSSM}} (1 + \delta_{\text{EW}}^{\text{lf}}) + \sigma_{gg\phi, \text{NNLO}}^t - \sigma_{gg\phi, \text{NLO}}^t. \quad (37)$$

SusHi leaves it up to the user to decide whether to use Eq. (31) or Eq. (37) in order to include the electro-weak corrections. For a SM-like Higgs and $m_\phi < 2m_t$, both approaches lead to comparable NLO results. The EW corrections for a CP-odd Higgs are not known and thus not included.

4. Cross section for bottom-quark annihilation

In supersymmetric theories, where the Higgs coupling to bottom-quarks can be enhanced by $\tan\beta$, associated production $(b\bar{b})\phi + X$ can be similarly or even more important than gluon fusion. Two theoretical approaches have been pursued for the theoretical description of this process: In the four-flavour scheme (4FS), the relevant production processes at lowest order QCD are $gg \rightarrow (b\bar{b})\phi$ (see Fig. 3 (a)) and quark-antiquark annihilation $q\bar{q} \rightarrow (b\bar{b})\phi$ [96–98]. However, when integrating over all final-state bottom-quark momenta, potentially large logarithms $\ln m_b/m_\phi$ occur. They can be resummed by the introduction of bottom-quark PDFs, which defines the five-flavour scheme (5FS) [99, 100]. The LO process in this latter scheme is bottom-quark annihilation $b\bar{b} \rightarrow \phi$ for which the lowest-order Feynman diagram is shown in Fig. 3 (b).

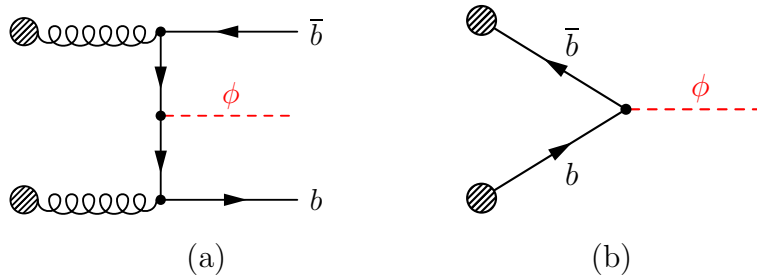


Figure 3: Feynman diagrams showing the associated production process $gg \rightarrow (b\bar{b})\phi$ in the 4FS (left) and bottom-quark annihilation in the 5FS (right).

SusHi implements results for associated $b\bar{b}\phi$ -production in the 5FS. For the inclusive cross section, it links the program **bbh@nnlo** [49] in order to obtain the NNLO QCD prediction σ_{bbH}^{SM} using $m_b^{\overline{\text{MS}}}(\mu_R)$ for the bottom Yukawa coupling. This is then re-weighted by the corresponding resummed SUSY coupling \tilde{g}_b^h [58, 101] as follows

$$\sigma_{bb\phi}^{\text{MSSM}} = \sigma_{bbH}^{\text{SM}} \cdot (\tilde{g}_b^\phi)^2 \quad \text{with} \quad \tilde{g}_b^h = \frac{g_b^h}{1 + \Delta_b} \left(1 - \Delta_b \frac{\cot\alpha}{\tan\beta} \right), \quad (38)$$

$$\tilde{g}_b^H = \frac{g_b^H}{1 + \Delta_b} \left(1 + \Delta_b \frac{\tan\alpha}{\tan\beta} \right), \quad \tilde{g}_b^A = \frac{g_b^A}{1 + \Delta_b} \left(1 - \Delta_b \frac{1}{\tan^2\beta} \right), \quad (39)$$

where the g_b^ϕ are given in Eq. (1) and Δ_b is determined as described in Section 2.3.

For differential cross sections due to bottom-quark annihilation, **SusHi** includes the NLO virtual corrections for $b\bar{b} \rightarrow \phi$ and combines them with the LO real-radiation processes $b\bar{b} \rightarrow g\phi$ and $bg \rightarrow b\phi$ using dipole subtraction.⁵ Similar to the fully inclusive case, we multiply with the resummed SUSY couplings to obtain MSSM cross sections.

⁵We are grateful to M. Wiesemann for providing us with the corresponding **Fortran** routines which entered the studies presented in Refs. [102, 103].

5. Differential cross sections

Apart from the total inclusive cross sections due to gluon fusion and bottom-quark annihilation, **SusHi** also allows for the computation of differential cross sections in these processes. In particular, one may apply upper and lower cuts on the Higgs' transverse momentum p_T , its rapidity y or its pseudo-rapidity η , where

$$\eta = -\ln \left(\tan \frac{\theta}{2} \right) = \frac{1}{2} \left(\frac{|\vec{p}| + p_L}{|\vec{p}| - p_L} \right), \quad y = \frac{1}{2} \left(\frac{E + p_L}{E - p_L} \right). \quad (40)$$

Here, $\vec{p} = \vec{p}_T + \vec{p}_L$ is the 3-momentum of the Higgs boson, p_L the longitudinal component, E the Higgs boson's energy, and θ the scattering angle (all in the hadronic reference frame). For gluon fusion, **SusHi** also provides the differential quantities $d\sigma/dp_T$, $d\sigma/dy$, and $d^2\sigma/(dp_T dy)$ (or, alternatively, $d\eta$ instead of dy). We add that, since the distribution in y and η is symmetric, minimal and maximal values for y are understood as $0 \leq y_{\min} \leq |y| \leq y_{\max}$ (and similarly for η). In order to get reliable results, the precision for the numerical integration in **SusHi** should be set to a higher value for differential quantities than for inclusive cross sections.

Note that at LO, i.e., $\mathcal{O}(\alpha_s^2)$ for gluon fusion and $\mathcal{O}(\alpha_s^0)$ for bottom-quark annihilation, the Higgs' transverse momentum is always $p_T = 0$. **SusHi** provides results for non-inclusive quantities through NLO, i.e., $\mathcal{O}(\alpha_s^3)$ for gluon fusion and $\mathcal{O}(\alpha_s)$ for bottom-quark annihilation. Let us also add that p_T -cuts or p_T -distributions should not be too low ($p_T/m_\phi \gtrsim 0.1$), since otherwise potentially large logarithms may spoil the perturbative convergence of the fixed-order results implemented in **SusHi**. For the resummation of such terms in Higgs production, see Refs. [104, 105], for example.

6. The program SusHi

This section describes the most important technical details of the program **SusHi**, including its installation and usage.

6.1. Workflow

The workflow of **SusHi** is depicted in Fig. 4. The input is controlled by a single input file whose format is SLHA-inspired [106, 107]. In case of the MSSM, the user specifies whether the Higgs mass is calculated by **FeynHiggs** or provided by the user himself. After the initialization of internal parameters which are derived from the input data, **SusHi** transforms them to the specified renormalization scheme and determines the resummation of $\tan \beta$ -enhanced terms in the bottom Yukawa coupling, see Sections 2.2.2 and 2.3. Afterwards, the gluon fusion and bottom-quark annihilation cross sections are calculated up to the desired perturbative order. The NNLO top-(s)quark induced and the electro-weak contributions for gluon fusion are taken into account only for the inclusive cross section. Not shown in the workflow is the link to **LHAPDF** which occurs at various stages of the internal calculation. The output is printed to the screen and written to an output file which follows the same format as the input file. Details concerning the in- and output files are given in Section 6.4.

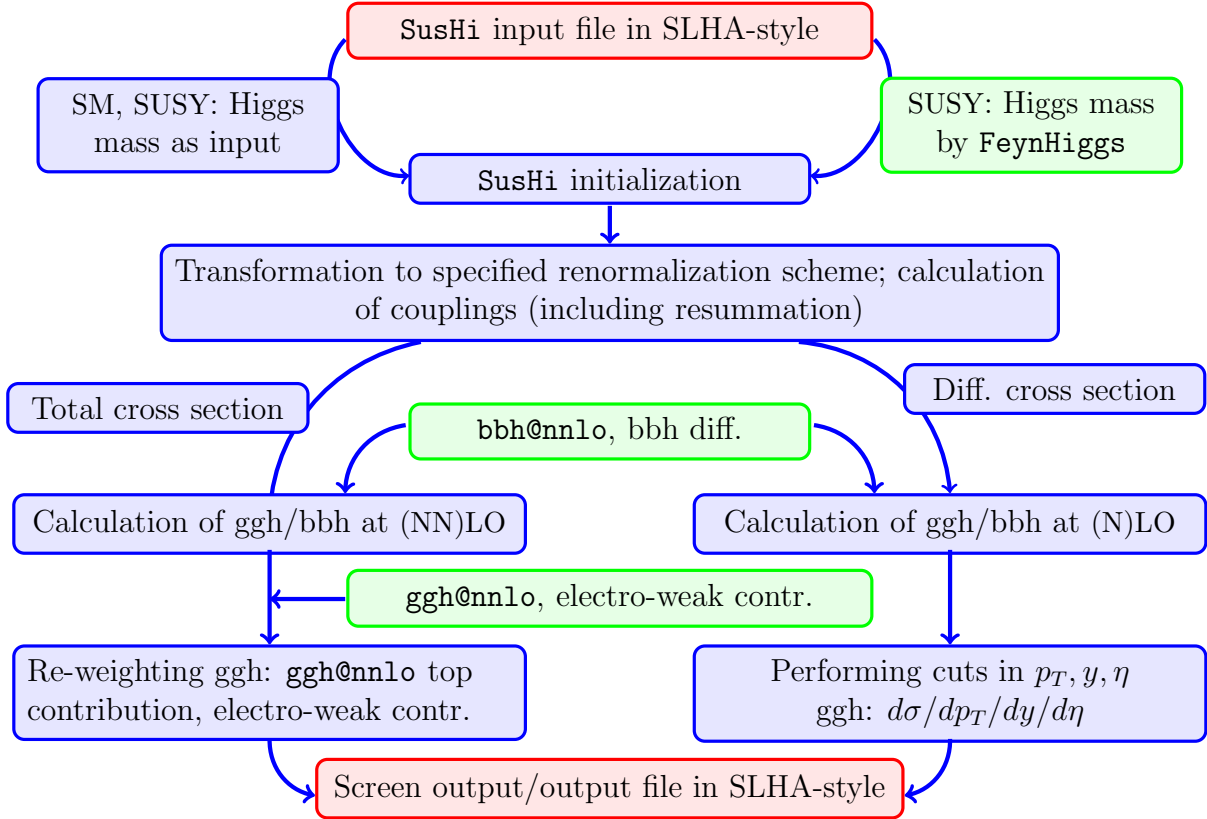


Figure 4: Internal workflow of **SusHi**. Red boxes indicate interaction with the user, who has to provide an input and gets an output file, if no error messages are shown. Green boxes refer to external code (see text), which is linked to/included in **SusHi**.

6.2. External code

As already mentioned, **SusHi** includes existing code like **ggh@nnlo**, **bbh@nnlo**, or the electro-weak grid. The integration of these programs does not require any action from the user though; they are part of the distribution and are simply linked to **SusHi** upon compilation.

However, **SusHi** can/must be linked to the following external code:

- **FeynHiggs** [62–65]: For the calculation of the supersymmetric Higgs masses $\phi \in \{h, H, A\}$, **SusHi** can be linked to **FeynHiggs**. Its input is controlled via the **SusHi** input files. Note that the current version of **SusHi** does not support complex MSSM parameters, so **FeynHiggs** is called with the default flags for the real MSSM.
- **LHAPDF** [61]: **SusHi** has to be linked to **LHAPDF** which provides a large variety of different PDF sets. This allows one to change the PDF set used by **SusHi** simply by changing the input file.

6.3. Installation and Usage

A tarball with the source files of **SusHi** can be obtained from Ref. [1]. Unpacking results in a main folder with the following subfolders:

bin : contains the executable program **sushi** after compilation

example : various example input files to be used in **bin**

include, **lib** : object files and libraries

src : **SusHi** source files, including external code

The file **README** in the main directory contains installation instructions and the history of the code. Compilation is most easily done by adjusting and running the Makefile in the main folder as follows:

- Specify the location and the name of the library of **LHAPDF**, for example:
PDFLIBP = /usr/local/lib
PDFLIB = -lLHAPDF
- If **FeynHiggs** should be used, specify the main directory of the compiled **FeynHiggs** library, for example:
FHPATH = /home/.../FeynHiggs-x.x.x
Note that **SusHi** requires **FeynHiggs** version 2.9 or higher.
- Run the configure script in the main folder:

```
./configure
```

It tries to find the **gfortran** or **ifort** compiler and the dependencies on your local machine. If you prefer a different compiler, or if the script fails, you can specify the relevant variables (**F77** and **LDLAGS**) in the file **compilerissues** yourself.

- In the main folder, run make:

```
make [option]
```

This command takes either of two optional arguments: **make** **predef=NoFeynHiggs** omits the link to **FeynHiggs** which cannot be used in this case; **make** **clean** deletes all object files, libraries, and the executable **sushi**.

After compilation, one may perform a test run of **SusHi** by copying one of the input files from the **example-** to the **bin-**folder, change to the **bin-**folder, and run

```
./sushi sushi.in sushi.out
```

Note that apart from the input filename, the user also has to provide a name for the output file. For parameter scans, we recommend the auxiliary routines **SLHAroutines**, which can be downloaded from Ref. [108]. In the following we will discuss the input and output files in more detail.

6.4. Input and output files

SusHi allows for calculations in the SM as well as in the MSSM. Although the former is in many respects a limiting case of the latter, SusHi distinguishes both cases, and we will discuss them separately in what follows.

6.4.1. Standard Model

A typical input file for a SM calculation is shown in the following. It divides into blocks, each of which contains a number of entries, specified by one or more leading blanks, an integer “entry number”, a value, and a comment initiated by the hash symbol #.

```

Block SUSHI
 1 0 # model: 0 = SM, 1 = MSSM
 2 0 # 0 = scalar Higgs (h), 1 = pseudoscalar Higgs (A)
 3 0 # collider: 0 = p-p, 1 = p-pbar
 4 8000.d0 # center-of-mass energy in GeV
 5 2 # order ggh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO
 6 2 # order bbh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO
 7 1 # electroweak cont. for ggh:
    # 0 = no, 1 = light quarks at NLO, 2 = SM EW factor
Block SMINPUTS # Standard Model inputs
 1 1.27934000e+02 # alpha_em^(-1)(MZ) SM MSbar
 2 1.16637000e-05 # G_Fermi
 3 1.17200000e-01 # alpha_s(MZ) SM MSbar
 4 9.11876000e+01 # m_Z(pole)
 5 4.20000000e+00 # m_b(m_b)
 6 1.73300000e+02 # m_t(pole)
 8 1.27500000e+00 # m_c(m_c)
Block MASS
 1 125.000000d0 # Higgs mass
Block DISTRIB
 1 0 # distribution : 0 = sigma_total, 1 = dsigma/dpt,
    # 2 = dsigma/dy, 3 = d^2sigma/dy/dpt
    # (values for pt and y: 22 and 32)
 2 0 # pt-cut: 0 = no, 1 = pt > ptmin, 2 = pt < ptmax,
    # 3 = ptmin < pt < ptmax
21 30.d0 # minimal pt-value ptmin in GeV
22 100.d0 # maximal pt-value ptmax in GeV
 3 0 # rapidity-cut: 0 = no, 1 = Abs[y] < ymax,
    # 2 = Abs[y] > ymin, 3 = ymin < Abs[y] < ymax
31 0.5d0 # minimal rapidity ymin
32 1.5d0 # maximal rapidity ymax
 4 0 # 0 = rapidity, 1 = pseudorapidity
Block SCALES
 1 1.d0 # renormalization scale muR/mh
 2 1.d0 # factorization scale muF/mh
Block RENORMBOT # Renormalization of the bottom sector
 1 0 # m_b used for bottom Yukawa: 0 = OS, 1 = MSbar(mb), 2 = MSbar(muR)
Block PDFSPEC
 1 MSTW2008lo68cl.LHgrid # name of pdf (lo)
 2 MSTW2008nlo68cl.LHgrid # name of pdf (nlo)
 3 MSTW2008nnlo_asmzrange.LHgrid # name of pdf (nnlo)
 4 0 # set number
Block VEGAS
 1 10000 # number of points
 2 5 # number of iterations
 3 10 # print: 0 = no output, 1 = prettyprint, 10 = table
Block FACTORS
 1 0.d0 # factor for yukawa-couplings: c
 2 1.d0 # t
 3 1.d0 # b

```

Block SUSHI specifies the crucial input for **SusHi**, namely the model, the kind of Higgs boson to be considered (scalar or pseudo-scalar⁶), the type of collider, the center-of-mass energy, the perturbative order for gluon fusion and bottom-quark annihilation, and to which extent electro-weak corrections to gluon fusion should be taken into account.

Block SMINPUTS contains the relevant SM input. We use the electro-magnetic coupling α_{EM} , Fermi's constant G_F , and the Z -boson mass m_Z (entries 1,2,4) to calculate the W mass m_W and the weak mixing angle $\sin \theta_W$. The input value for $\alpha_s(m_Z)$ given in entry 3 is used for renormalization-group (RG) running and RG transformations. We allow this value to be different from the one required by the PDFs which are specified further below. The latter is taken from **LHAPDF** and enters the calculation as the coupling parameter of the perturbative expansion of the cross section, for example Eq. (19) and Eqs. (21). The charm- and bottom-quark masses (entries 8,5) are to be given in the $\overline{\text{MS}}$ scheme as $m_c(m_c)$ and $m_b(m_b)$, while the top-quark mass (entry 6) is required in the on-shell scheme. In the SM, the Higgs mass is a free parameter and has to be provided in **Block MASS**, entry 1.

Block DISTRIB controls cuts or distributions with respect to the transverse momentum p_T , the (pseudo-)rapidity y (η), if desired. Note that differential cross sections (entry 1 $\in \{1, 2, 3\}$) can only be obtained for gluon fusion; in this case, entries 22 and/or 32 specify the value of p_T and/or y (η). Cuts are possible both for gluon fusion and bottom-quark annihilation; they are applied by setting entry 1 to 0 ("total cross section"), and specifying entries 2,21,22 and/or 3,31,32. Note that entry 4 changes between rapidity y and pseudo-rapidity η .

Block SCALES defines the renormalization and factorization scales relative to the Higgs mass. In accordance with Section 2.3, **Block RENORMBOT** offers different options for the renormalization of the bottom Yukawa coupling; three options are currently implemented: $m_b^Y \in \{m_b^{\text{OS}}, m_b^{\overline{\text{MS}}}(m_b), m_b^{\overline{\text{MS}}}(\mu_R)\}$. **Block PDFSPEC** contains the PDF sets in the notation of **LHAPDF**, consisting of the name of the PDF grid file, and the set number. **Block VEGAS** specifies integration parameters; note that distributions or cuts require higher numerical precision than the total cross section in order to reach comparable accuracy in the final result. Finally, **Block FACTORS** allows for additional factors in the Yukawa couplings of the fermions. We add that also charm-quark contributions can be taken into account by setting the corresponding factor to 1. Then the c -quark contributions at (N)LO are added using the on-shell value m_c^{OS} calculated from $m_c(m_c)$ as done for the on-shell bottom-quark mass. In case of the MSSM, for which a detailed prescription follows, the charm-quark contributions can be added as well.

⁶Apart from the scalar Higgs boson of the actual SM, **SusHi** also provides results for a pseudo-scalar Higgs-like particle whose coupling to fermions is obtained from the corresponding MSSM couplings by setting $\tan \beta = 1$.

6.4.2. Minimal Supersymmetric Standard Model

In case of the MSSM, the input file contains a number of additional Blocks. We show them here, together with the relevant modifications of the SM version:

```
Block SUSHI
 1 1 # model: 0 = SM, 1 = MSSM
 2 1 # 0 = light Higgs (h), 1 = pseudoscalar (A), 2 = heavy Higgs (H)
[.....]
 5 2 # order ggh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO, 3 = ~NNLO stop for h
Block MINPAR # SUSY breaking input parameters
 3 5.d0 # tanb
Block EXTPAR
 3 800.d0 # M_3
11 2006.66d0 # A_t
12 2006.66d0 # A_b
23 200.d0 # mu in GeV
26 300.d0 # M_A0
43 1000.d0 # M_Q3
46 1000.d0 # M_TR
49 1000.d0 # M_BR
Block FEYNHIGGS # FeynHiggs specific input
 1 0.d0 # M_1
 2 200.d0 # M_2
 3 2006.66d0 # A in GeV (except for A_t, A_b)
 4 1000.d0 # M_SUSY in GeV (except for M_Q3, M_TR, M_BR)
Block RENORMBOT # Renormalization of the bottom sector
 1 0 # m_b used for bottom Yukawa: 0 = OS, 1 = MSbar(m_b), 2 = MSbar(muR)
 2 1 # tan(beta)-res. of Y_b: 0 = no, 1 = naive, 2 = full (for OS only)
 3 1 # Delta_b: Take Delta_b from FeynHiggs: 0 = no, 1 = yes
Block RENORMSBOT # Renormalization of the sbottom sector
 1 2 # m_b: 0 = OS, 1 = DRbar, 2 = dep; recommended: 2
 2 0 # A_b: 0 = OS, 1 = DRbar, 2 = dep; recommended: 0
 3 0 # theta_b: 0 = OS, 1 = DRbar ; recommended: 0
Block FACTORS
 1 0.d0 # factor for yukawa-couplings: c
 2 1.d0 # t
 3 1.d0 # b
 4 1.d0 # st
 5 1.d0 # sb
```

Entry 2 of Block SUSHI now distinguishes between the three MSSM Higgs bosons. Entry 5 allows to add approximated NNLO stop contributions for the light Higgs h . Block MINPAR, entry 3 defines the value of $\tan\beta$. Block EXTPAR fixes the parameters of the third family of quarks and squarks in the MSSM. If the Block FEYNHIGGS is present, SusHi has to be linked to FeynHiggs (see Section 6.3) which will then be used to calculate the Higgs masses from the parameters of that Block. In addition to the SM version, Block RENORMBOT provides various ways of resumming t_β -enhanced effects for the on-shell bottom Yukawa coupling, see Section 2.3. For a running coupling (entry $1 \in \{1, 2\}$), resummation of those effects is always performed as shown in Eq. (16f). Block RENORMSBOT provides the choice between the various options of Tab. 1. The alternative to Block FEYNHIGGS is the specification of the Higgs masses and the Higgs mixing angle α by hand; for example:

```
Block ALPHA
-2.58961078E-01 # mixing in Higgs sector
Block MASS
25 125.216431E+00 # Higgs mass h
26 303.288802E+00 # Higgs mass H
36 130.000000E+00 # Pseudoscalar Higgs mass A
```

In this case, it is the user's responsibility to assure consistency of the Higgs mass and the other parameters. However, this option allows one to use `H3m` [77], for example, in order to take into account three-loop effects to the SUSY Higgs mass [76–78].

Three example input files can be found in the subfolder `example`, namely a SM input file and two MSSM input files, the latter two for the usage with and without `FeynHiggs`.

6.4.3. Output file

`SusHi` outputs the results of the calculation as well as some key parameters derived from the input in the same format as the input file. A typical example is shown here:

```
Block SUSHiggh # Bon appetit
      1      1.65830627E+01      # ggh XS in pb
Block SUSHibbh # Bon appetit
      1      3.83211336E-01      # bbh XS in pb
Block XSGGH # ggh MSSM-Cross sec. in pb
      2      1.21882475E+01      # NLO
[.....individual channels]
Block XSGGHEFF # ggh MSSM-Cross sec.
      1      1.44925736E+01      # ggh@NLO SM
      2      1.82888376E+01      # ggh@NNLO SM
      3      5.78917317E-02      # electroweak factor
Block XSBBH # bbh MSSM-Cross sec. in pb
      1      5.30129833E-01      # LO
      2      4.79035691E-01      # NLO
      3      3.83211336E-01      # NNLO
Block HGSUSY # couplings of light Higgs h to 3. generation
[.....]
Block MASSOUT
      5      4.20000000E+00      # m_b(m_b), MSbar
      25     1.25216431E+02      # MSSM-Mh in GeV
[.....SM masses/sbottom/stop masses]
Block ALPHA # Effective Higgs mixing parameter
      -2.58961078E-01      # alpha
Block STOPMIX # stop mixing matrix
      1  1      7.07918788E-01      # V_11
      1  2     -7.06293841E-01      # V_12
      2  1      7.06293841E-01      # V_21
      2  2      7.07918788E-01      # V_22
[.....]
Block AD
      3  3      2.00666000E+03      # used Ab in GeV - def. accord. to scheme
Block AU
      3  3      2.00666000E+03      # used At in GeV
```

The main result for the gluon fusion cross section, containing all corrections specified by the user in the input file, is given as entry 1 in Block `SUSHiggh`, the one for bottom-quark annihilation in Block `SUSHibbh`. Individual contributions to the cross sections are listed in Block `XSGGH` and Block `XSBBH`; their meaning should be obvious from the comment in the output file. Note that the results denoted “LO” etc. mean that the LO partonic cross section is convolved with the PDF set given in entry 1 of Block `PDFSPEC` in the input file.

For gluon fusion, the Block `XSGGHEFF` contains the NNLO top-(s)quark results as obtained by `ggh@nnlo`, and the electro-weak correction factor as determined in Section 3.4.

In addition, Block `HGSUSY` lists the non-resummed MSSM couplings of the quarks and squarks to the Higgs boson under consideration. Block `MASSOUT` gives the relevant SM and SUSY masses as well as the Higgs mass. Not shown above is the Block `INTERNALMASSES`,

which provides the different bottom masses entering the calculation of gluon-fusion cross sections, and **SCALESOUT** showing the renormalization and factorization scale as well as the value of $\alpha_s(\mu_R)$ taken from the PDF set at (N)LO. Finally all output files have the corresponding input file attached at the end.

7. Conclusion

In this article we described the **Fortran** code **SusHi** for the calculation of the cross section for Higgs production in gluon fusion and bottom-quark annihilation at hadron colliders. It works both in the SM and the MSSM, evaluates inclusive cross sections, distributions, and allows for kinematical cuts on the Higgs 4-momentum. It includes higher order QCD and electro-weak corrections and takes into account the effect from squarks and gluinos.

SusHi allows one to choose among various renormalization schemes for the sbottom sector and the bottom Yukawa coupling, and includes the resummation of $\tan\beta$ -enhanced effects. For the calculation of the Higgs mass in the MSSM **SusHi** can be linked to **FeynHiggs**. **SusHi** can be downloaded from Ref. [1].

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Appendix A. Formulas: Higgs-squark couplings

In this section we present the couplings of the three neutral Higgs bosons ϕ of the MSSM to the quarks and squarks being implemented in **SusHi**. The relevant Feynman rules can be written in the form

$$\begin{array}{c} \phi \text{ (dashed red line)} \end{array} \begin{array}{c} \nearrow \bar{q} \\ \searrow q \end{array} = i \frac{m_q}{v} g_q^\phi \quad \text{and} \quad \begin{array}{c} \phi \text{ (dashed red line)} \end{array} \begin{array}{c} \nearrow \tilde{q}_j \\ \searrow \tilde{q}_i \end{array} = i \frac{m_q^2}{v} g_{\tilde{q},ij}^\phi, \quad (\text{A.1})$$

where $v = 2m_W/g = 1/\sqrt{\sqrt{2}G_F} = \sqrt{v_d^2 + v_u^2}$. The couplings g_q^ϕ of the Higgs boson ϕ to the quarks q with respect to the SM Higgs boson coupling were already presented in Eq. (1). The couplings $g_{\tilde{q},ij}^\phi$ of the squarks to the light and heavy Higgs can be split in the form

$$g_{\tilde{q},ij}^\phi = g_{\tilde{q},ij}^{\phi,\text{EW}} + g_{\tilde{q},ij}^{\phi,\mu} + g_{\tilde{q},ij}^{\phi,\alpha}. \quad (\text{A.2})$$

In case of the light Higgs h we obtain for the couplings:

$$g_{\tilde{t},11}^{h,\text{EW}} = c_{\tilde{t},1}^{\text{EW}} c_{\theta_{\tilde{t}}}^2 + c_{\tilde{t},2}^{\text{EW}} s_{\theta_{\tilde{t}}}^2 \quad g_{\tilde{b},11}^{h,\text{EW}} = c_{\tilde{b},1}^{\text{EW}} c_{\theta_{\tilde{b}}}^2 + c_{\tilde{b},2}^{\text{EW}} s_{\theta_{\tilde{b}}}^2 \quad (\text{A.3})$$

$$g_{\tilde{t},22}^{h,\text{EW}} = c_{\tilde{t},1}^{\text{EW}} s_{\theta_{\tilde{t}}}^2 + c_{\tilde{t},2}^{\text{EW}} c_{\theta_{\tilde{t}}}^2 \quad g_{\tilde{b},22}^{h,\text{EW}} = c_{\tilde{b},1}^{\text{EW}} s_{\theta_{\tilde{b}}}^2 + c_{\tilde{b},2}^{\text{EW}} c_{\theta_{\tilde{b}}}^2 \quad (\text{A.4})$$

$$g_{\tilde{t},12}^{h,\text{EW}} = g_{\tilde{t},21}^{h,\text{EW}} = \frac{1}{2} (c_2^{\text{EW}} - c_1^{\text{EW}}) s_{2\theta_{\tilde{t}}} \quad g_{\tilde{b},12}^{h,\text{EW}} = g_{\tilde{b},21}^{h,\text{EW}} = \frac{1}{2} (c_2^{\text{EW}} - c_1^{\text{EW}}) s_{2\theta_{\tilde{b}}} \quad (\text{A.5})$$

$$g_{\tilde{t},11}^{h,\mu} = -g_{\tilde{t},22}^{h,\mu} = \frac{\mu}{m_t} \frac{\cos(\alpha - \beta)}{s_\beta^2} s_{2\theta_{\tilde{t}}} \quad g_{\tilde{b},11}^{h,\mu} = -g_{\tilde{b},22}^{h,\mu} = -\frac{\mu}{m_b} \frac{\cos(\alpha - \beta)}{c_\beta^2} s_{2\theta_{\tilde{b}}} \quad (\text{A.6})$$

$$g_{\tilde{t},12}^{h,\mu} = g_{\tilde{t},21}^{h,\mu} = \frac{\mu}{m_t} \frac{\cos(\alpha - \beta)}{s_\beta^2} c_{2\theta_{\tilde{t}}} \quad g_{\tilde{b},12}^{h,\mu} = g_{\tilde{b},21}^{h,\mu} = -\frac{\mu}{m_b} \frac{\cos(\alpha - \beta)}{c_\beta^2} c_{2\theta_{\tilde{b}}} \quad (\text{A.7})$$

$$g_{\tilde{t},11}^{h,\mu} = \frac{c_\alpha}{s_\beta} \left(2 + \frac{m_{\tilde{t}1}^2 - m_{\tilde{t}2}^2}{2m_{\tilde{t}}^2} s_{2\theta_{\tilde{t}}}^2 \right) \quad g_{\tilde{b},11}^{h,\mu} = -\frac{s_\alpha}{c_\beta} \left(2 + \frac{m_{\tilde{b}1}^2 - m_{\tilde{b}2}^2}{2m_{\tilde{b}}^2} s_{2\theta_{\tilde{b}}}^2 \right) \quad (\text{A.8})$$

$$g_{\tilde{t},22}^{h,\mu} = \frac{c_\alpha}{s_\beta} \left(2 - \frac{m_{\tilde{t}1}^2 - m_{\tilde{t}2}^2}{2m_{\tilde{t}}^2} s_{2\theta_{\tilde{t}}}^2 \right) \quad g_{\tilde{b},22}^{h,\mu} = -\frac{s_\alpha}{c_\beta} \left(2 - \frac{m_{\tilde{b}1}^2 - m_{\tilde{b}2}^2}{2m_{\tilde{b}}^2} s_{2\theta_{\tilde{b}}}^2 \right) \quad (\text{A.9})$$

$$g_{\tilde{t},12}^{h,\mu} = g_{\tilde{t},21}^{h,\mu} = \frac{c_\alpha}{s_\beta} \frac{m_{\tilde{t}1}^2 - m_{\tilde{t}2}^2}{2m_{\tilde{t}}^2} s_{2\theta_{\tilde{t}}} c_{2\theta_{\tilde{t}}} \quad g_{\tilde{b},12}^{h,\mu} = g_{\tilde{b},21}^{h,\mu} = -\frac{s_\alpha}{c_\beta} \frac{m_{\tilde{b}1}^2 - m_{\tilde{b}2}^2}{2m_{\tilde{b}}^2} s_{2\theta_{\tilde{b}}} c_{2\theta_{\tilde{b}}} \quad (\text{A.10})$$

Therein we made use of the abbreviations $s_x = \sin x$ and $c_x = \cos x$ and defined:

$$c_{\tilde{t},1}^{\text{EW}} = -\frac{m_Z^2}{m_{\tilde{t}}^2} \left(1 - \frac{4}{3} s_{\theta_W}^2 \right) \sin(\alpha + \beta) \quad c_{\tilde{b},1}^{\text{EW}} = \frac{m_Z^2}{m_{\tilde{b}}^2} \left(1 - \frac{2}{3} s_{\theta_W}^2 \right) \sin(\alpha + \beta) \quad (\text{A.11})$$

$$c_{\tilde{t},2}^{\text{EW}} = -\frac{m_Z^2}{m_{\tilde{t}}^2} \frac{2}{3} s_{\theta_W}^2 \sin(\alpha + \beta) \quad c_{\tilde{b},2}^{\text{EW}} = \frac{m_Z^2}{m_{\tilde{b}}^2} \frac{2}{3} s_{\theta_W}^2 \sin(\alpha + \beta) \quad (\text{A.12})$$

The couplings to the heavy Higgs H are easy to obtain by the replacement $\alpha \rightarrow \alpha - \frac{\pi}{2}$ in the previous formulas. In case of the CP-odd Higgs A the couplings are given by:

$$g_{t,11}^A = g_{t,22}^A = g_{b,11}^A = g_{b,22}^A = 0 \quad (\text{A.13})$$

$$g_{t,12}^A = -g_{t,21}^A = \frac{1}{t_\beta} \frac{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}{2m_t^2} s_{2\theta_{\tilde{t}}} + \frac{\mu}{m_t} \left(1 + \frac{1}{t_\beta^2} \right) \quad (\text{A.14})$$

$$g_{b,12}^A = -g_{b,21}^A = t_\beta \frac{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}{2m_b^2} s_{2\theta_{\tilde{b}}} + \frac{\mu}{m_b} (1 + t_\beta^2) \quad (\text{A.15})$$

We add that m_b is partially interpreted as the bottom mass in the sbottom sector, namely where it is meant to be part of the Higgs-sbottom coupling.

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